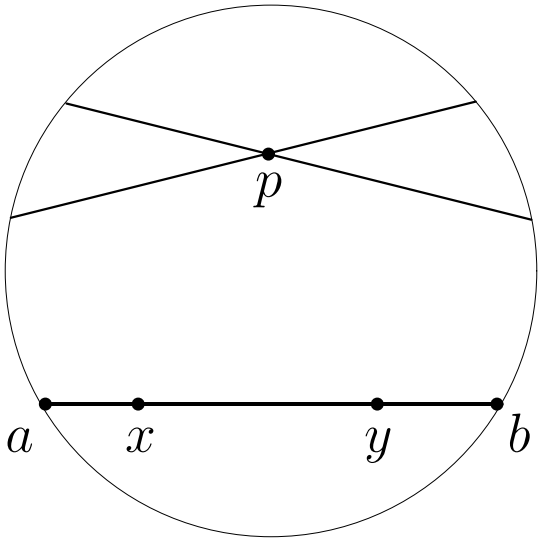
Hilbert Geometries Workshop

CIRM, Luminy

9 to 12 January, 2012

Organizers: B. Lemmens and C. Vernicos



1 Program

Monday		
09.15-10.15	M. Troyanov	Hilbert geometry as a canonical geometry
10:45-11-45	L. Marquis	Quotient of Hilbert Geometry
15.00-16.00	B. Lemmens	Birkhoff's version of Hilbert's metric and its applications in analysis
16.30-17.30	J. Lawson	The Thompson metric and an axiom system for symmetric cones
Tuesday		
09.15-10.15	A. Beardon	The hyperbolic and other metrics in complex analysis
10:45-11-45	S. Gaubert	Nonexpansive maps, zero-sum games and tropical convexity
14.00-15.00	A. Papadopoulos	Isometries of the Funk and the Hilbert metric
15.30-16.30	C. Walsh	Horofunctions and isometries of Hilbert geometries
17.00-18.00		OPEN PROBLEM SESSION
Wednesday		
09.15-10.15	A. Thompson	Monotonicity of area in (non-symmetric) normed spaces
10:45-11-45	J.C. Alvarez-Paiva	Integral geometry on Finsler manifolds
15.00-16.00	M. Akian	Fixed points and eigenvectors of convex monotone dynamical systems
16.30-17.30	Y. Lim	A deterministic approach to the least squares mean on Hadamard spaces
Thursday		
09.15-10.15	X. Nie	The Hilbert geometry of simplicial Tits sets
10:45-11-45	M. Crampon	About volume entropy
14.00-15.00	P. Verovic	Hilbert domains that admit a quasi-isometric embedding into Euclidean space
15.30-16.30	C. Vernicos	Approximability and volume growth of Hilbert geometries

Meals at CIRM

Breakfast from 07.00 to 09.00.

Lunch at 12:30.

Dinner at 19:30.

2 Titles & Abstracts

Fixed points and eigenvectors of convex monotone dynamical systems

Marianne Akian
(INRIA and CMAP École Polytechnique Palaiseau, France)

Abstract: Convex, order preserving maps of $\mathbb{R}^n$ coincide with the dynamic programming operators of stochastic control problems with n states and discrete time, hence their fixed points are value functions of infinite horizon stochastic control problems. Such maps that are additively homogeneous correspond to undiscounted control problems, and they are necessarily nonexpansive for the additive Hilbert semimetric. Moreover, in the particular deterministic case, they are max-plus linear. We shall present several results concerning the structure of the sets of fixed points or eigenvectors of all these maps, generalizing max-plus spectral theory. We shall also present some infinite dimensional generalizations in the particular max-plus linear case using the max-plus Martin boundary.

This talk covers joint works with Stéphane Gaubert, Bas Lemmens, and Cormac Walsh.

∞

Integral geometry on Finsler manifolds

Juan-Carlos Alvarez-Paiva
(University of Lille, France)

Abstract: Finsler geometry is the natural framework for many problems and constructions in integral geometry. In this talk I'll stress the basic concepts and geometric construction while concentrating on Hilbert geometries and projective Finsler spaces.

∞

The hyperbolic and other metrics in complex analysis

Alan Beardon
(Cambridge University, United Kingdom)

Abstract: We shall discuss the role of the hyperbolic and other metrics in complex analysis, especially as a source of conjectures in other situations.

∞

About volume entropy

Mickaël Crampon
(University of Santiago de Chile, Chile)

Abstract: I want to talk about the volume entropy conjecture, which states that the volume entropy of a n-dimensional Hilbert geometry is always smaller than $n-1$, that is, the entropy of hyperbolic space. A (a priori) more difficult problem consists in finding the geometries whose entropy attains the conjectured maximum.

The conjecture is completely proved in dimension 2 by Berck, Bernig and Vernicos who also showed that if the boundary is of class $C^{1,1}$, then entropy equals $n-1$. I managed to understand more about the strictly convex divisible cases (those which admit a compact quotient) using dynamics of the geodesic flow. This is basically what is known.

In the talk, I would like to propose a possible approach to the problem, inspired by the work of Berck, Bernig and Vernicos and the entropy theory of hyperbolic systems. The goal is to make possible an open discussion on the question, which would benefit from the various specialities of each of us.

∞

Nonexpansive maps, zero-sum games and tropical convexity

Stephane Gaubert
(INRIA and CMAP École Polytechnique Palaiseau, France)

Abstract: Shapley operators (dynamic programming operators of zero-sum games) can be identified with non-linear order preserving self-maps of the standard positive cone. The latter maps are nonexpansive in Hilbert's and Thompson's metric. In fact, a number of results concerning repeated games (asymptotic behavior of the value function as the horizon tends to infinity), with a Denjoy-Wolff flavor, can be approached by metric geometry techniques, exploiting in particular the horoboundary in the Funk hemi-metric. I will discuss here some generalizations of two classical results (the Kohlberg-Neyman theorem for nonexpansive mappings in Banach spaces, the Collatz-Wielandt theorem in Perron-Frobenius theory) obtained along these lines, as well as some relations which have recently emerged with tropical geometry (tropical convex sets).

This talk is based on a joint work with G. Vigeral (arXiv:1012.4765) and on a joint work Akian and Guterman (arXiv:0912.2462).

∞

The Thompson Metric and an Axiom System for Symmetric Cones

Jimmie Lawson
(Louisiana State University, United States)

Abstract: We introduce an elementary axiom system for a symmetric cone, valid for the finite and infinite dimensional case. The axioms involve overlaying an algebraic symmetric structure on the interior of a normal cone in a Banach space. One can deduce the Busemann seminegative curvature of the Thompson metric in this general setting from the assumption of a weak version of the geometric-arithmetic mean inequality, as well as obtaining other standard inequalities familiar from operator theory. As a special case, one obtains that the symmetric cone from a JB-algebra satisfies a certain convexity property for the Thompson part metric: the distance function between points evolving in time on two geodesics is a convex function.

∞

Birkfoff's version of Hilbert's metric and its applications in analysis

Bas Lemmens
(University of Kent, United Kingdom)

Abstract: In the 1950s Garrett Birkhoff introduced a version of Hilbert's metric on cones, and combined it with the contraction mapping principle to prove a number of results in the spectral theory of linear operators that leave a cone in a Banach space invariant. In recent years this idea has been further developed. It has led to a number of strikingly detailed results for nonlinear operators on cones. In this talk I will give a survey of these results and discuss some related open problems concerning nonexpansive mappings on Hilbert geometries

∞

A deterministic approach to the least squares mean on Hadamard spaces

Yongdo Lim
(Kyungpook National University, Korea)

Abstract: We propose a fast and deterministic approach to the least squares mean on Hadamard spaces.

A simple and direct proof of some important properties of the least squares mean, which have been established by Sturm using probabilistic methods on the metric structure of Hadamard spaces, is provided.

We illustrate our approach by applying it to compute the Karcher mean on the convex cone of positive definite matrices which plays a central role in the study of inversion invariant data averaging procedures in image processing and in radar detection.

∞

The Hilbert geometry of simplicial Tits sets

Xin NIe
(University Pierre and Marie Curie, France)

Abstract: The simplest examples of divisible convex sets, called "Tits sets", are those generated by reflections with respect to a polytope in real projective space. In the case where the polytope is a simplex, we will present a "inverse triangle inequality", which enables us to obtain many informations about the Hilbert metric in this case.

∞

Monotonicity of area in (non-symmetric) normed spaces

Anthony Thompson
(Dalhousie University, Canada)

Abstract: After reviewing the wide variety of possible area functions on a finite-dimensional normed space the talk will look at the requirement of monotonicity – that larger metrics generate larger areas – and explore some of the immediate consequences. In the final section we will consider how this requirement might apply in the non-symmetric case.

∞

Hilbert Geometry as a Canonical Geometry

Marc Troyanov
(Ëcole Polytechnique Fédéral de Lausanne, Switzerland)

Abstract: There is at first view no Riemannian metric that we can canonically attach to a convex domain in the real projective space and that would be complete and satisfy some natural geometric properties. However one can introduce a canonical Finsler metric on such a domain and this metric coincides with the metric defined by Hilbert in 1895. It is a natural metric in the sense of being invariant under projective transformations and its geodesics are the straight lines. In this lecture which is meant to be introductory, we will explain the basic notions of Finsler geometry, construct the Hilbert metric as a canonical Finsler metric and compute some examples. We will also see some geometric properties of these metrics and discuss Hilbert's fourth problem.

∞

Approximability and volume growth of Hilbert geometries

Constantin Vernicos
(University of Montpellier 2, France)

Abstract: I will relate the volume entropy of a Hilbert geometry to another invariant of its boundary, called the approximability. This will allow me to give a proof of the optimal upper bound on the volume entropy in dimension 2 and 3.

∞

Hilbert domains that admit a quasi-isometric embedding into Euclidean space

Patrick Verovic
(University of Savoie, France)

Abstract: We prove that a Hilbert domain which admits a quasi-isometric embedding into a finite-dimensional normed vector space is actually a convex polytope.

∞

Horofunctions and isometries of Hilbert geometries

Cormac Walsh
(INRIA and CMAP Ecole Polytechnique Palaiseau, France)

Abstract: The horofunction boundary of a metric space provides a useful tool for studying isometries of the space. In this talk, I describe the horofunction boundary of the Hilbert geometry. I also give an account of joint work with Bas Lemmens, where we use this knowledge to determine the isometry group when the domain of the Hilbert geometry is polyhedral.

∞

3 List of participants

1. C. Vernicos (Montpellier, FR), constantin.vernicos@math.univ-montp2.fr
2. B. Lemmens (Canterbury, GB), B.Lemmens@kent.ac.uk
3. M. Akian (Palaiseau, FR), Marianne.Akian@inria.fr
4. J.C. Alvarez-Paiva (Lille, FR), Juan-Carlos.Alvarez-Paiva@math.univ-lille1.fr
5. A. Beardon (Cambridge, GB), A.F.Beardon@dpmms.cam.ac.uk
6. B. Colbois (Neuchâtel, CH), bruno.colbois@unine.ch
7. M. Crampon (Santiago de Chile, CL), crampon@math.u-strasbg.fr
8. P. Foulon (Luminy, FR), foulon@cirm.univ-mrs.fr
9. S. Gaubert (Palaiseau, FR), Stephane.Gaubert@inria.fr
10. J. Lawson (Baton Rouge, US), lawson@math.lsu.edu
11. Y. Lim (Taegu, KR), ylim@knu.ac.kr
12. L. Marquis (Rennes, FR), ludovic.marquis@univ-rennes1.fr
13. X. Nie (Paris, FR), nie@math.jussieu.fr
14. A. Papadopoulos (Strasbourg, FR), papadop@math.u-strasbg.fr
15. A. Thompson (Halifax, CA), tony@mathstat.dal.ca
16. M. Troyanov (Lausanne, CH) marc.troyanov@epfl.ch
17. P. Verovic (Chambéry, FR), verovic@univ-savoie.fr
18. C. Walsh (Palaiseau, FR), cormac.walsh@inria.fr
19. Q. Zheng (Palaiseau, FR), quzheng2003@gmail.com